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Centralized and Decentralized Stochastic Routing Models in Telecommunication Networks

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ABSTRACT

With the advent of non-hierarchical routing in circuit-switched telecommunication networks, on-line routing policies have been developed with the objective of optimizing some measure of gain or performance. These policies are decentralized. However, traditional planning models are centralized models. We present a decentralized routing model to be used in network planning. We compare it theoretically and empirically with a centralized multicommodity flow model previously presented. The two models are solved by the same type of algorithm, a convex simplex implementation, adapted differently according to the model. Comparative results between planning models reproducing the two policies are discussed.

Keywords: routing models, centralized optimization, decentralized optimization, circuit-switched networks, network performance.

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RÉSUMÉ

Les nouveaux commutateurs utilisés dans les réseaux à commutation par circuits ont rendu possible l'introduction de politiques d'acheminement d'appels non-hiérarchiques qui optimisent, en temps réel, certaines mesures de performance. Ces politiques sont décentralisées. Cependant, les modèles traditionnels de planification de réseaux sont du type centralisé. Nous présentons un modèle décentralisé d'acheminement d'appel à être utilisé dans la planification. Nous le comparons, théoriquement et empiriquement, avec un modèle centralisé présenté auparavant. Les deux modèles ont été résolus avec le même algorithme, un simplexe convexe, adapté différemment dans chaque cas. Nous discutons les résultats obtenus.

Mots-clés: acheminement d'appels, optimisation centralisée, optimisation décentralisée, réseaux à commutation par circuits, performance de réseaux.

1 Introduction

Telecommunications is one of the technological areas which have undergone dramatic changes in recent years. In fact, the introduction of new technologies in this field has changed the whole philosophy of telecommunication network design, call routing and network management. In particular, switching techniques have become much more flexible and non-hierarchical routing is being introduced. Traditional routing policies, such as hierarchical routing, being simple rules that determined where to place a call when it arrived, did not explicitly consider routing optimization. However, the introduction of a new switching technology, such as switches with stored program control and new signalling techniques, provides more information about the state of the system thus allowing a new flexibility to in routing calls. This flexibility has made it feasible to consider optimizing the routing with respect to some measure of performance.

From an optimization standpoint, two types of routing models are possible: centralized and decentralized models. In a centralized scheme there is a central entity that places the calls in the most efficient way in order to optimize the global performance of the network. Conversely, in decentralized schemes there are local entities that try to optimize their local performance. From a performance point of view, centralization is clearly more efficient than decentralization. However, in order to explain why centralization models are not widely used, we need to give more details about routing models and routing technology.

Routing can be generally defined as the problem of how to place a call when it arrives. A routing policy is, therefore, a set of rules needed to assign the arriving calls. We call this problem *on-line routing*. For many years, the only on-line routing policy available was *hierarchical* routing (see Wilkison 1956). The advent of modern switches with stored program capabilities made it feasible to create routing rules based on the state of the system. However, for planning purposes, it is necessary to know in advance, for a given network, policy and demand matrix, how the demand will be assigned to the different links or routes. The assignment can vary according to the different performance or cost measures that the planner wants to consider. Mathematical models having as objective the optimization of those measures are thus designed. We call those models *static planning routing models* to underline the difference with on-line routing.

Centralized optimization models are very suitable as static planning models; they give rise to elegant multicommodity formulations that allow the planner to solve large scale problems by using powerful algorithms. The flaw that has been traditionally found in those formulations is that they do not represent reality. In fact, even though the routing techniques are evolving from the simply hierarchical networks toward more sophisticated non-hierarchical routing, we are still far from having a totally centralized policy. Centralized models are thus mostly used for planning purposes, sometimes as comparative basis for on-line policies.

The purpose of this paper is to introduce a new type of decentralized static planning routing model for circuit-switched networks, closer to reality than classical centralized optimization multicommodity formulations, that may replace such models at the planning stage. We compare the model, theoretically and empirically, with a centralized scheme with a similar objective function formulated as a multicommodity flow problem.

The paper is organized as follows. In section 2 a literature review on the issues of centralization vs decentralization is presented. In section 3 the most important features of the centralized optimization model are recalled. The decentralized model is introduced in section 4. The theoretical relationship between the two models is outlined in section 5 whereas the models are compared on the basis of a computational study in section 6. Comparative performance results and concluding remarks are presented in section 7 and 8, respectively. An appendix has been added to clarify some technical aspects of the presentation.

2 Literature Review

The notions of centralized and decentralized routing optimization have been considered, for a long time, by the traditional contraposition between system and user optimization that can be found in transportation (see, for instance Wardrop (1956), Dafermos and Sparrow (1969)). In telecommunication networks, routing optimization is a more recent notion. In fact, in circuit switched networks, hierarchical routing, which was conceived mainly for administrative purposes, was the only available policy. The rules, that determined what to do with an arriving call, did not raise the issue of optimization either centralized or decentralized. The first rigorous approach toward optimized routing was presented by Benes (1966) which presented the basis of the Markov decision theory modeling that has allowed modern on-line routing technology to include the optimization of some parameters.

Centralized optimization (also called system-optimized) routing models were first considered in packet-switched networks, due in part to the flexibility of the routing for such networks. Fratta, Gerla and Kleinrock (1973) formulated the centralized optimization static routing problem as a convex programming problem and introduced the flow deviation algorithm, which is an adaptation of the Frank-Wolfe algorithm used to solve convex multicommodity formulations. Gallager (1977) followed the same trend suggested by Fratta et al, but for quasi-static routing problems. Also in packet-switching, Gavish and Hantler (1983) proposed a centralized non-linear combinatorial optimization problem to tackle the problem of route selection in a computer network while Mason(1985) described analytic models for decentralized adaptive routing and computed off-line the centralized optimization routing patterns.

In the area of circuit-switching, Mason and Girard (1982) presented an unifying framework for a number of routing techniques and evaluated performance models by comparing them with exact Markov chain analysis; they also discussed the issues of centralized and decentralized optimization. More recently, Kelly (1988) presented a centralized model based on revenue maximation and showed how the existence of shadow prices associated to the routes of the network could be used as a basis for a decentralized, on-line adaptive routing scheme.

3 The Centralized Optimization Model

In this section, we introduce the stochastic centralized optimization model. This model was first described in Sansó (1988) and a particular application of it was presented in Sansó, Soumis and Gendreau (1991); however, since many modelling details are common with the decentralized model and for the sake of completeness in the theoretical and empirical comparison between the two models, its most relevant features will be briefly recalled. The reader interested in further details can find them in the previously mentioned references.

In the routing schemes that are found in the literature (see, for instance Girard (1990), Dziong et al (1988), Krishnan (1982), Mason and Girard (1982), Deschamps (1979), Benes (1966)), the optimization of the network performance is usually an explicit or implicit objective. A performance measure commonly used is the global probability of blocking in the network that in most cases is determined by Erlang's B formula. This function is non-convex with respect to the traffic variable, and therefore the use of this formula in the objective of a centralized optimization problem over the traffic variables leads to a non-convex problem, which is very difficult to solve. One common approach, then, was to optimize the routing with respect to a cost function, generally a linear function, and explicitly add a set of performance constraints. In the case of hierarchical routing, the performance constraints corresponded to "grade of service" constraints, i.e. constraints on the blocking probability on the final trunk of a route. With the advent of non-hierarchical routing, the notion of "final trunk" has been lost, and the grade of service requirements have being replaced by a set of recursive non-linear constraints stating that a compound measure related to the global probability of blocking in the system be inferior to a specific value (see Girard (1990), chapter 4). Solving the problem with such constraints are very time consuming.

For this reason, in Sansó (1988), a different approach was taken: the lost call traffic was chosen as a measure of network performance and such a measure was optimized directly, not added as a set of constraints.

For the reader to understand the philosophy of the model, we first present its general characteristics. There is a demand, in terms of calls that have to be connected, from every origin to every destination. This demand of calls from different origin-destination

pairs of the network will compete for the capacity available. If the demand is such that the calls cannot get through the network, the calls will simply be lost. In the centralized or cooperative scheme of this present model, the competition is managed by a central entity. Calls are routed in a way such as to minimize the number of calls that are lost, globally, in the network.

In a deterministic context, this problem is a multicommodity flow problem with capacity constraints. Such problems are extremely difficult to solve (for a wide review on multicommodity problems, see Assad (1978)). One technique to solve those problems consists in eliminating the capacity constraints and adding, instead, a penalty term to the objective function. In this way, the problem becomes a easy to solve nonlinear problem. This was exactly the approach proposed by Fratta, Gerla and Kleinrock in their work done on packet-switched networks. In their case, the capacity constraints were replaced by a convex delay penalty function.

In circuit-switched networks, there is no notion of delay, but rather the notion of blocking. However, as it was stated earlier on, the probability of blocking would not have been a good penalty function, due to its non-convex nature. Sansó (1988) proposed the expected lost call traffic as a convex penalty function. This not only allows the capacity constraints to be dropped, thus converting a difficult problem into an easy one, but also measures a factor that one would naturally want to minimize in a communication network. This function, Erlang's lost call traffic function, has the additional advantage of taking into account the stochastic character of the demand. Therefore, the problem becomes much easier to solve than the original one, and is closer to reality than one where artificial penalties are added.

We now introduce some necessary notation:

N: node set for the entire graph;
M: arc set for the entire graph;
O: origin node set;
C_u: capacity of arc u;
X_u^p: traffic offered to arc u that comes from origin p.

Let $B(s,a)$ be the well known Erlang's blocking probability function, which gives

the probability of blocking a call on a link with a capacity of s lines and a demand, or offered traffic, of a Erlangs. The expression for this blocking probability formula is given by:

$$B(s,a) = \frac{a^s/s!}{\sum_{i=1}^{s} a^i/i!}. \tag{1}$$

This formula can also be seen as representing the proportion of calls that will be blocked, as a function of the capacity and the link offered traffic. This function has been proven convex with respect to variable s (see Messerli(1972),Jagers and Van Doorn(1986)) and the non-convexity of the function with respect variable a has been studied by Harel (1990). Let now $P_u(X)$ be the penalty function chosen, that is Erlang's expected lost call traffic function. This function can be expressed as a function of the probability of blocking as follows:

$$P_u(X) = \sum_{p \in O} X_u^p \; B(C_u, \sum_{p \in O} X_u^p \;). \tag{2}$$

Note that, since only the X variables intervene in the routing model, the capacity variables being fixed, the function above has been denoted only in terms of the offered traffic, X.

Let Ω represent the set of flows such that flow conservation and nonnegativity constraints are respected. Then the centralized optimization model (COM) can be formulated as follows,

COM:

$$\min_{X \in \Omega} Z = \sum_{u \in M} P_u(X). \tag{3}$$

The model introduces two approximations, the first one is the flow conservation of the offered traffic. This is not true in networks with blocked-calls-lost policies. The real model should be in that case a generalized multicommodity flow problem with a non-linear gain function. To our knowledge, no algorithm has ever been developed to tackle such problems. The decentralized model introduced later in this paper will not use

this approximation, the flow conservation constraints will be on the accepted traffic. The second approximation introduced by the model is the fact that the objective function is additive and separable by arc. This function implicitly relies on the assumption that the probability that a call is blocked in more than one arc is negligible. In other words, this objective function is an upper bound of the total lost call traffic of the network. The bound is closer to the actual value when there is little congestion in the network. As flow increases, the difference between the actual value and the bound increases as well; at the same time the optimal solution will display a more balanced structure en terms of link flows. In other words, if the real function were optimized, the actual value of the lost call traffic would be smaller but a more extremist solution will be found, i.e. some links would carry light traffic in detriment of other, very congested, links.

The objective function of model COM is convex since the link lost call traffic function $P(X)$ over variable X is convex (this was first shown by Palm (1957) and Sansó (1988) derived a different proof). This insures the existence of a unique (with respect to the link flows) optimal solution to the model and guarrantees the convergence of any descent algorithm.

4 The Decentralized Model

In the decentralized model presented in this section, the network is not seen as a cooperative entity in which all calls are routed in order to optimize the global performance but rather as made up of several "markets", each having its local decision center, that will compete for the capacity available in order to route their calls. This competing behavior will lead towards an equilibrium in the system. We have used the term *market* in this context to emphasize the idea of competition in the decentralized model. A market, can be, in fact, any origin-destination pair of nodes of the system.

Let us now describe the routing features of the model. Let R be the set of markets in the system. Then, for every market $r \in R$, there is a choice of n routes to assign the demand of the market, d^r. The possible routes that can be taken for a particular market r are represented in the set Q_r. The demand of the market, will be assigned to *all* routes of the market according to an attraction coefficient A_q such that:

$$\sum_{q \in Q_r} A_q = 1. \tag{4}$$

The portion of the demand that is refused, tries again. This makes the model closer to reality than the centralized model, where the calls that could not be routed were simply lost. The refused portion of the demand it is thus re-assigned on the n routes according to the route attraction coefficients. This re-assignement is performed *regardless* of the route a particular call tried the previous time. In the model, the lost call traffic is treated in an explicit manner, whereas all the other factors, such as preference of a route due to its length, importance, etc. will be included in the choice of the attraction coefficients. In the appendix we explain how the attraction coefficients chosen in our model correspond to the solution of a quadratic optimization problem.

In order to be able to express the model behavior in mathematical terms, we need some additional notation:

U_q: set of arcs belonging to route $q \in Q_r$;
X_q: offered traffic to route $q \in Q_r$;
H_q: traffic accepted on route $q \in Q_r$;

$\mathcal{X}$: vector representing the offered traffic on all routes of the network;
H: vector representing the accepted traffic on all routes of the network;
$\mathcal{P}_q(\mathcal{X})$: lost call traffic on route $q \in Q_r$ as a function of the network offered traffic $\mathcal{X}$.

Under the behavior described for the decentralized model, variable X_q, which represents route q offered traffic, must be proportional to the attraction coefficient given to the route, A_q and there must be a variable u_r for every $q \in Q_r$ such that

$$X_q = u_r A_q, \quad \forall r \in R. \tag{5}$$

One also has

$$X_q = H_q + \mathcal{P}_q(\mathcal{X}). \tag{6}$$

The offered traffic on route q is split in accepted traffic and overflow. We now present the relationship between the overflow on route q, $\mathcal{P}_q(\mathcal{X})$, and the lost call trafffic on a particular arc of the route.

Let X_u^r represent the offered traffic of market r that is trying to pass through arc $u \in U_q$ and let X_u represent the total offered traffic on arc u, that is:

$$X_u = \sum_{r \in R} X_u^r. \tag{7}$$

The overflow for route q can then be expressed as:

$$\mathcal{P}_q(\mathcal{X}) = \sum_{u \in U_q} (X_u^r / X_u) P_u(\mathcal{X}). \tag{8}$$

This equation implicitly relies on the same assumption made for model (COM), that is, that the probability of having a call blocked more than once in the same route is

negligible. In addition, the equation holds only if one supposes that there is no discrimination with respect to the call nature for the routing or blocking of a call: the blocking probability of a particular call is independent of its nature, i.e. its origin/destination.

Let us suppose that it is possible to find a function of the accepted traffic that is the inverse of the network offered traffic, that is:

$$s(H) = \mathcal{X}, \tag{9}$$

then the decentralized model (*DOM*) can now be formulated in terms of the accepted traffic variables.

(DOM):

$$\left.\begin{array}{l} \dfrac{H_q + \mathcal{P}_q(s(H))}{A_q} = u_r, \\ \displaystyle\sum_{q \in Q_r} H_q = d^r. \end{array}\right\} \forall r \in R. \tag{10}$$

In the next section we will explore the possibility of solving this model with the same type of algorithm that is used to solve the centralized model.

5 Relationship between the Centralized and the Decentralized model

One way of solving the system of nonlinear equations describing the decentralized model (DOM) is by assuming that those equations may correspond to the optimality conditions of some centralized optimization problem. This approach, which has been widely used to solve user-optimized models in transportation (see, for instance Dafermos and Sparrow (1969)), would be of particular interest for our purpose of comparing the two types of models. In fact, if model (DOM) had some equivalent centralized optimization model, the same solution algorithm could be used to solve (COM) and (DOM) and, therefore, the result comparison would be facilitated.

In our particular problem, the question we have to ask is if there exists an objective $\mathcal{F}$ such that

$$\frac{\partial \mathcal{F}}{\partial H_q} = \frac{H_q + \mathcal{P}_q(s(H))}{A_q}. \tag{11}$$

According to the Fondamental Theorem of Calculus this objective function $\mathcal{F}$ will exist if the following condition is respected:

$$\frac{\partial^2 \mathcal{F}}{\partial H_q \partial H_p} = \frac{\partial^2 \mathcal{F}}{\partial H_p \partial H_q}. \tag{12}$$

This condition is equivalent to

$$A_p \frac{\partial \mathcal{P}_q(s(H))}{\partial H_p} = A_q \frac{\partial \mathcal{P}_p(s(H))}{\partial H_q}. \tag{13}$$

When routes p and q are such that they do not have any arcs in common, then the partial derivatives of (13) are null and the condition holds. However, when p and q have arcs in common, the derivatives are non-zero and (13) can be re-written as:

$$\frac{\partial \mathcal{P}_q(s(H))}{\partial H_p} \Big/ \frac{\partial \mathcal{P}_p(s(H))}{\partial H_q} = \frac{A_q}{A_p}. \tag{14}$$

Since the attraction coefficients are chosen independently of the lost call traffic, there is no reason for (14) to hold. Therefore, for the general case, model (DOM) *does not* have an equivalent centralized optimization problem whose optimality conditions is the set of nonlinear equations defined by (10). However, the conditions do hold *within* a market where the routes are link disjoint. For reliability reasons, it is advisable for the routes belonging to the same market to be link-disjoint. Therefore, for such as case, it exists a function $\mathcal{F}$ such that the subproblems (COPr) and (DOPr) that follow are equivalent.

(COPr):

$$\min \mathcal{F} \tag{15}$$

s.t.

$$\sum_{q \in Q_r} H_q = d^r. \tag{16}$$

(DOPr):

$$\frac{H_q + \mathcal{P}_q(s(H))}{A_q} = u_r, \tag{17}$$

$$\sum_{q \in Q_r} H_q = d^r. \tag{18}$$

The equivalence of the two subproblems constitutes the theoretical basis for the cyclic decomposition algorithm used to solve the decentralized model (DOM).

6 Resolution

In this section, we first describe the general algorithm that was used to solve both models. This is followed by explanations on the specific implementation for each model, in particular, the decomposition procedure that allowed us to solve the decentralized model (DOM).

6.1 The Algorithm

The algorithm used to solve the optimization problems is an adaptation of the Convex Simplex classical method, first presented by Nguyen(1973) and implemented by Sauvé (1984). It takes advantage of the commodity separation and the network structure of the problem and has a special criterion for the entering variable.

Firstly, the variables of the basis are grouped by commodity with each group becoming a spanning tree of the network. Thus, no matrix storage is required for the simplex iterations and only the tree belonging to the entering variable commodity will be used in every iteration.

Secondly, the entering variable is not chosen as the one with the most negative reduced cost. Instead, a positive ϵ_1 is initially set and the entering variable is selected among those variables, of a particular commodity k, whose reduced cost is inferior to $(-\epsilon_1)$.

At each step, all commodities are optimized and ϵ_1 is reduced by a rate λ, $(\lambda < 1)$. When $\epsilon_1 \leq \epsilon_2$, ϵ_2 being fixed in advance, and no improved solution can be found for any commodity, then the procedure terminates.

The gradual reduction of ϵ_1 prevents the algorithm from spending too much time on one commodity if the other commodities have not yet been adjusted. However, the value given to λ allows us to modify the algorithm behavior. In fact, if λ is close to 1, ϵ_1 will decrease very slowly and the algorithm will behave as a traditional Convex Simplex. On the other hand, if λ is small, ϵ_1 will soon become very close to the stopping criteria ϵ_2 which will make the algorithm spend a long time on the optimization of one

particular commodity. In other words, the algorithm, for this case, will behave as a decomposition algorithm. Therefore, a calibration of λ and ϵ_1 is necessary to get an efficient intermediate algorithm.

6.2 The Computational Procedure

Model (COM) was solved with a straightforward adaptation of the Convex Simplex algorithm described. For this model, the commodities were represented by the origin nodes. The variables λ and ϵ_1 were set so that the procedure behaved as an algorithm intermediate between a traditional Convex Simplex and a decomposition. The convergence of the procedure is guaranteed by the convexity of the objective function of model (COM).

For the second model, the same algorithm was used as a block Gauss-Seidel procedure. The network was divided in markets, every market representing a commodity. λ was set small to make the algorithm behave as a decomposition. The procedure, which can then be seen as a cyclic decomposition scheme, adjusted at every stage the variables for each market r, so that the conditions defined by subproblem (DOPr) were reached. The procedure was repeated until no significant change was detected in any of the markets.

The convergence of the Gauss-Seidel method for a set of nonlinear equations is not guaranteed. Ortega and Rheinboldt (1970) have established some conditions to be respected that would guarantee the convergence of the method, however they are very difficult to verify in practice. In general two types of conditions can be identified: a certain symmetry of the Jacobian matrix of the system of equations defined as $f(X) = 0$, f being a convex function (see Hammond and Magnanti (1987)), or the diagonal dominance of the Jacobian matrix, even if f is a non-convex function (see Ortega and Rheinboldt). The first condition guarantees a convergence of the objective function of an equivalent optimization problem. The second condition guarantees a convergence on the variables of the problem, even if an equivalent optimization problem does not exist. It was shown, in the previous section, that the set of equations defining model DOM has no equivalent optimization problem. Therefore we tackle the convergence problem on the basis of the second type of condition. The condition could not be theoretically verified for the general model. In fact, the conditions depends on the the problem data

and it will always be possible to create a problem so that the Gauss Seidel algorithm will not converge or will have multiple solutions.

The problem of multiple solutions of a system of non-linear equations describing a particular routing model or phenomenon it is not a new problem in circuit switched networks. We mention, for instance, the works by Nakagome and Mori (1973) in which it is shown that the system of equations describing the exact flow values of a mutual overflow network can present multiple solutions. Akinpelu (1983) showed that networks with non-hierarchical alternate routing can have bi-stable modes of operation. The stability of circuit switched networks in systems with mutual overflow and non-hierarchical routing was studied by Mason (1986). The study related various parameters to network instability for three different types of non-hierarchical routing models. One of the characterizations of network instability was the non-uniqueness of equilibrium flow patterns. The study concluded that instability is a pervasive phenomenon in networks with mutual overflow and provided means to stabilize otherwise unstable networks.

Despite the fact that convergence conditions could not be verified for a general network, oscillatory or divergent behavior can easily be detected during the solution procedure. One may thus apply a modified version, which checks for such behavior, of the previously defined algorithm: this method will either terminate with an equilibrium solution or a non-convergent condition. In general, the oscillatory behavior will take place when there is a strong interaction between markets, in other words, when mutual overflow is important.

7 Results

The network used to evaluate the two types of models was a hypothetical network of 40 nodes and 62 arcs. The implementation for the centralized model required the handling of 5960 variables and 40 commodities. The implementation of the decentralized model needed the handling of 4800 variables and 1600 commodities.

The behavior of the centralized and the decentralized models was evaluated for different levels of demand. These different levels of demands were obtained by multiplying the original demand matrix by factors between 50% and 150%. The figure depicts the lost call traffic for different levels of demand, for the two models.

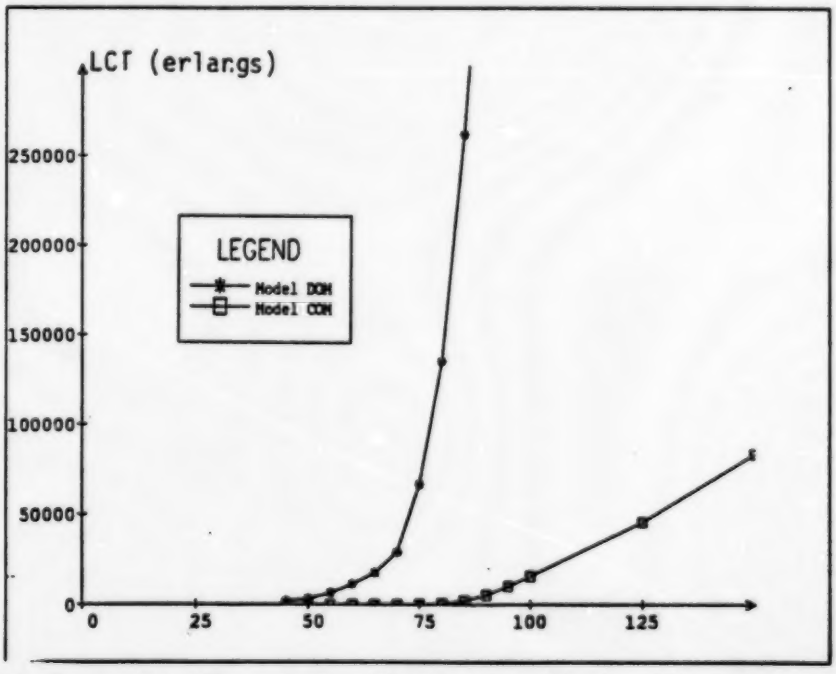

Figure 1: Lost call traffic vs demand for the centralized and decentralized models

The figure shows that for small levels of demand, both models are almost equivalent. On the other hand, the decentralized model reaches an important level of congestion

much earlier than the centralized model. It can be seen that the decentralized model presents a vertical asymptote when the network is too congested. In other words with a decentralized type of routing such as the one considered, the network could not handle an important increase on the level of the demand. On the other hand, the centralized model also presents an asymptotical behavior, but toward a line of slope 1. This is due to the fact that, for greater levels of demand the assumption made regarding the additivity of the objective function of model (COM) is no longer valid. In fact, if it were possible to formulate model (COM) in terms of the accepted traffic rather than the offered traffic, the model would present a vertical asymptote like the one obtained for model (DOM), but this time, the asymptote would occur for higher levels of congestion.

The comparison between model (COM) and model (DOM), is therefore particularly valid for intermediate levels of congestion, that is if the values of variables X are H are similar.

In table 1 we present total execution times for the two models for different levels of demand. The times given are total execution times in seconds on a SUN 3/50 workstation.

TABLE 1

Level of demand (%)	time (COM) (sec)	time (DOM) (sec)
50	12.2	667
55	12.9	729
60	13.9	853
65	14.9	998
70	17.7	1190
75	20.2	1301
80	29.2	1437
85	37.2	1675
90	46.7	2086
95	46.9	1866
100	38.7	1862
125	41.7	2100
150	44.5	2173

As expected, we observe that the decentralized model takes a little bit more than 40 times more time to solve than the centralized one. This is due to the fact that the decentralized model has been formulated in terms of origin- destination pairs as being the commodities whereas the centralized one was formulated in terms of the origin nodes as being the commodities.

8 Summary and Conclusions

We presented a new type of decentralized model that is closer to reality than centralized models and that could be used in network planning. We compared it, on the basis of theoretical and empirical considerations, with a centralized optimization model. The solution approaches correspond to two different implementations of the same Convex Simplex type algorithm. The centralized optimization model was solved directly, whereas a cyclic decomposition procedure was created to solve the decentralized model by the Gauss-Seidel method.

Both models were solved quickly on a SUN 3/50, for the same empirical application. However, as could be expected, the decentralized model was more than 40 times longer to solve than the centralized one. This difference in the solution times can be traced back to the fact that the first model can be formulated as having one commodity per origin whereas the second must be formulated in terms of individual origin-destination pairs.

The two models were evaluated for a changing level of demand. The performance of the network, in terms of how well it could handle congestion for a particular routing policy, was the measure of comparison. As expected, it was found that the centralized optimization model is more efficient in terms of network performance than the decentralized one. The most important and unexpected result is the significant error introduced when using a centralized planning model in congested network when actual routing policies are decentralized. However, when the levels of congestion are low, such in the new fiber optic networks, the centralized model constitutes an efficient tool to evaluate network performance.

The objective of any routing policy is to optimize the management of calls on the network and this has an impact, not only on network performance, but on network dimensioning as well. The centralized approach was already incorporated into an algorithm to optimize Network Dimensioning based on the routing policy and reliability considerations. We refer the interested reader to Sansó, Gendreau and Soumis (1990). It would be interesting to develop a similar algorithm that would take into account the decentralized planning model presented in this paper. Significantly different planning decisions (dimensioning) would probably be obtained.

Appendix

There are several ways to choose the attraction coefficients so that equation (4) was satisfied. We wanted to include the route lenght in the coefficients, so that the product of the lenght by the flow in every route was minimized. This would have led to a linear programming formulation, which would have meant to have very extremist solutions. Such solutions, grouping on the same path the total flow of some origin-destination pair, are not acceptable from a reliability point of view. The alternative, thus, was to choose the closest non-linear programming formulation, i.e. a quadratic one:

$$\min_{X} \sum_{r \in R} \sum_{q \in Q_r} X_q^2 L_q \tag{19}$$

s.t.

$$\sum_{q \in Q_r} X_q = d^r \quad \forall r \in R. \tag{20}$$

L_q represented the length of a particular route $q \in Q_r, r \in R$.

The Kuhn-Tucker optimality conditions of this problem are:

$$X_q = u_r / L_q \qquad \forall q \in Q_r, \forall r \in R, \tag{21}$$

This quadratic model induces optimality conditions similar to equation (5) in which $(1/L_q)$ corresponds to the attraction coefficient A_q. If we make a normalization so that the sum of all the coefficients of the same route is equal to one (equation (4)), we obtain:

$$A_q = \frac{\mathcal{L}_q}{\sum_{i \in Q_r} \mathcal{L}_i} \qquad \forall q \in Q_r, \forall r \in R. \tag{22}$$

where, $\mathcal{L}_k = (1/L_k)$. Note that according to the precedent equations, the attraction coefficient of a route is proportional to the inverse of its lenght, which is consistent with our objective.

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